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Abstract - This paper explores general equilibrium consumption choices and interest rate determination in a two-period model in which the production side explicitely describes the thermodynamic process unavoidably connected with production, as argued by Georgescu Roegen. A simple energy based production process is modeled, which is not in a stationary state. The resulting production function is time dependent. In neoclassical general equilibrium the thermodynamic implication of the production process, i.e. the production of waste, will not be taken into account by decision making agents. For welfare optimality, the resulting externality need be corrected by a social planner, or through the use of environmental related taxation. However, it is shown that imposing in the same economy energy as a medium of exchange (money), makes agents "energy conscious" and decreases the externality associated with entropic waste production through a market mechanism, without the need for the intervention. In the limit case in which production occurs in thermodynamic equilibrium, no entropic waste is produced, and the model collapses to the nested neoclassical model. A contribution of the proposed approach is the determination of energy (money) prices in general equilibrium. Despite the fact that energy does not enter the agents utility function, and therefore has no direct value, money prices and interest rate can be fully characterized in the model due precisely to the production technology adopted. The change in the numeraire and medium of exchange performed affects the economy due to the non stationarity of the production process, but has no effect in the limit case in which the productive process reaches steady state.

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1 Introduction

Among the key objections to the welfare optimality of competitive market equilibrium, the issue of externalities, which are not properly evaluated by decision making agents, is a critical one. These negative consequences of the individuals decision making process impact general welfare but not enough the decision making individual. Since the work of Ayres and Kneese (1969), waste externalities and environmental pollution has acquired a central place among the negative externalities of the economic process. Environmental pollution can be broadly characterized as entropic waste, i.e. dispersion into the environment of low quality energy, creating environmental disorder without achieving any economically valuable goal (Kümmel (1989)).

Entropic pollution cannot generally be accounted for in a neoclassical general equilibrium model. In addition to the difficulties of highlighting the role of externalities in general, entropic pollution will not be present in such models in which the formal structure of production theory is based on stationary production functions. The stationary neoclassical production function does not accommodate the role of time and the related concepts of entropy and waste. These issues have been sharply analyzed in the work of Georgescu Roegen (1971), which provides the basis for a realistic general theory of production, containing the concept of waste. Nevertheless, the neoclassical paradigm of production is often used in the analysis of thermodynamic efficiency of production (Ruth (1995), Stern (1997)).

In this paper, production activity is described by means of a simple production function which makes completely apparent the underlying irreversible thermodynamic process, as suggested by Roma (2000). The implications for the equilibrium in the economy and waste production are fully spelled out through a simple deterministic model.

The structure of the firm's problem in respect of the level of waste is very similar to the theory presented in Ethridge (1973). The firm jointly produces two products, one of which can be split into an economically useful product and waste. However, here we include time to produce and a thermodynamic constraint on production which fully specifies the technology. Moreover, in the approach presented a physical limit to aggregate production possibilities is introduced, and the technology does not allow input substitution.

With this description of the production side, entropic pollution becomes visible, and we are able to formally characterize the associated waste in a general equilibrium framework. Still, in the neoclassical model decision making agents will completely ignore it in their choices. Producers will set optimal production plans ignoring the consequences of waste, and the affected consumers will not be individually able to influence this decision. However, it is shown that a simple monetary arrangement, namely adopting energy as numeraire, will lead decision making agents to modify their choices, compared to the pure barter economy solution, twisting the equilibrium in the direction of a decrease of the amount of entropic waste. Such choice of the numeraire is a natural way to make agents energy waste conscious and internalize the externality. The proposed mechanism provides an alternative to the usual approach to waste reduction through environmental related taxation. A contribution of the present approach is the determination of energy (money) prices in general equilibrium, which can be solved for due to the specific thermodynamic production process assumed. As a limiting case, if production indeed occurs in thermodynamic equilibrium (which would require infinite time), then entropic pollution is not produced (and need not be taken into account), and the resulting market equilibrium will revert to that of a neoclassical barter economy. The neoclassical competitive equilibrium model turns out to be a special case of the model proposed here. In this way, the model illustrate the connection between stationary and time dependent economic production, reconciling this approach with classical textbook microeconomics.

The plan of the paper is as follows: Section 2 describes the Georgescu Roegen type production process, and presents a simple thermodynamic characterization of technology. The production process takes place over a finite time interval, and allows for output to be available at two different dates, with the unavoidable joint production of waste. In Section 3 the tradeoff between output (and consumption) at the two dates is embedded in a simple general equilibrium model, and the various possibilities to internalize the waste externality are discussed. A feature of the production technology is that it allows easy computation of the product energy price. We can adopt energy as the numeraire of the economy, and compute the resulting decentralized equilibrium. Section 4 provides a discussion of the modifications of the decentralized equilibrium, with respect to the neoclassical solution, brought about by the use of the energy numeraire. In particular, it is shown that under the energy numeraire the production of entropic waste will be smaller than in the corresponding neoclassical barter economy. Therefore, in the presence of waste externalities, market equilibrium under the energy numeraire may deliver the same optimal allocation which would other ways be obtained in a centralized economy. Section 5 contains some concluding remarks.

2 Production

Production of the single good in the economy is achieved, at two different dates, by a representative firm. Production is achieved by "mixing", through the use of a non perishable production plant, two inputs, which are available in nature at no cost, but which are useless until properly combined through the production process. Shares of the firms are owned by individuals in the economy, and we assume, for simplicity, that cannot be traded. The inputs are thermal energy, and a row material (a fluid) which is available in unlimited quantity. In addition, we must mention the use of the plant.

Denoting energy E, the raw material X, and the plant L, we could try to simplify the description of production into a neoclassical production function $Y = f(E, X, L)$.

However, this has many limitations. According to Georgescu Roegen (1971, chapter IX, p. 234), the neoclassical production function is the equivalent of the list of ingredients at the top of a "cookbook recipe". It does not describe the sequence and the technique used to mix the ingredients together, i.e. the proper recipe. It does not say, among other things, *for how long* should the mixed inputs cook for. It only says that we can obtain the quantity z of product from the quantities $(x, y, \ldots)$ of the inputs, without any reference to the time sequence of the production process. Hence the production function as a point function, i.e. from a vector of real numbers to a single real number.

Among the shortfalls of the neoclassical production function approach there is the fact that it ignores i) time required to produce, ii) the qualitative change that materials undergo within the production process, and iii) the unavoidable production of waste in the process. The alternative representation of the production process proposed by Georgescu Roegen keys on the description of all the flows that enter and exit the process (cross its boundary) over time. Georgescu Roegen's forceful argument that no production process can really go into steady state, and that the laws of thermodynamics imply the unavoidable production of entropic waste is not accommodated by the neoclassical production function.

We take a longer route to describe the stylized production process in this economy, and properly represent the three characteristics of a production process just mentioned, which would be ignored by the use of a neoclassical production function. To this effect, we fully describe the time sequence of actions needed to achieve the finished product. We will later reconcile this approach with a neoclassical production frontier approach, to be used in the context of a general equilibrium model. This longer route will then prove to be fruitful in a number of ways. Among other results, the point is made that it is possible to accommodate the production of entropic waste in a general equilibrium model, by describing in finer detail the thermodynamic aspects of production.

2.1 The production process

The production process will consist, following Georgescu Roegen (1971), of flows of input into the process, and flows of outputs out of the process. We recall, and make use of, the observation by Georgescu Roegen (1971, p.216) that qualitative change is unavoidably associated with the production process, and that that some inputs are qualitatively changed by the production process in such a way that, when they exit the process, they are so different that need to be represented as a different output.[1]

The production process takes place continuously in the time interval $t \in [-1, 1]$. The input flows will be $E(t)$, $-1 \leq t \leq 1$, representing thermal energy which flows into the process throughout the period, and $X(t)$, $t = -1$, representing the (possibly very large) amount of row material which flows into the process only at the beginning of the period. The flows of output will be $Y(t)$, $t = 0$, $t = 1$, the finished product which is extracted from the plant at two points in time, and $W(t)$, $t = 1$, the amount

[1] See his examples, from a "rested worker" which exits the production process as a "tired worker", to sand, coal, et. which exit the furnace as melted glass. Examples are countless.

of waste that will be discarded into the environment at the end of the process. The production plant is, in Georgescu Roegen terminology, a "fund of services" (call it $L(t)$), which (forcing the argument slightly) remains unchanged inside the process throughout the time interval $t \in [-1, 1]$.

The single consumable good is obtained in this economy by the qualitative transformation of the raw material input, $X(t)$, by means of thermal energy. The output $Y(t)$ is nothing but the input $X(t)$ which has undergone an irreversible qualitative transformation. The simplest irreversible transformation we consider is the heating of the raw input, to a temperature greater or equal to k, by means of thermal energy. Such example is chosen in view of its analytical tractability, but in principle any transformation could be considered. It is clear that in the case examined here, the basic input undergoes a qualitative transformation. One of its qualities, its temperature, has changed. We assume that the transformation is irreversible, in the sense that it is not possible to subsequently recover, or unbundle, from the finished product the energy input used. This is a consequence of the second law of thermodynamics. Some of the energy transmitted to the raw material will increase the material's entropy, an will never be recovered. Moreover, as it will be shown, in finite time it will be impossible to produce a perfectly uniform product and waste will be jointly produced.

In this setting, we may represent production as the transformation of $X(t)$ into $Y(t)$ by describing the quality change of $X(t)$, namely its change in temperature. We will say that $X(t)$ is "finished product" if, due to the energy input, its temperature is greater or equal to k. In doing so, the need for a separate notation $Y(t)$, for the finished product, will be replaced by the temperature coordinate of the input $X(t)$.

2.2 Production Technology

We now describe the production process in greater detail.

Production of the good is achieved by the conduction heating of the input fluid X. The fluid, which is available in nature at a temperature, T_0, lower than k, is useless until is heated. A source of heat at (possibly time varying) temperature $T_1(t)$ is exogenously available to achieve this task. The initial raw input $X(t)$ is initially stored in the production plant, which may be visualized as a cylindric capacitor with unit base area. Depth inside the capacitor, denoted x, will measure the volume of input fluid (and eventually output). Once the upper surface of the full capacitor is in contact with a thermal energy source at temperature $T_1(t) > k > T_0$, the function $T(t, x, T_0, T_1)$ will describe the temperature of the input fluid at depth x at time $t \geq -1$. This function is positive, monotonically decreasing in x (and increasing in t), and has continuous first and second derivatives. Temperature $T(t, x, T_0, T_1)$ satisfies Fourier's equation

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k_E \frac{\partial T}{\partial x}\right], \ \rho, c, k_E > 0 \tag{1}$$

where k_E is the thermal conductivity with respect to energy, c is the fluid specific temperature coefficient of energy per unit of mass and ρ denotes mass density.

A *thermodynamic equilibrium* is achieved if temperature does not vary with time, that is, may be defined by

$$\frac{\partial T}{\partial t} = 0. \tag{2}$$

Finished product will be defined by the implicit constraint

$$T(t, x, T_0, T_1) \geq k.$$

This constraint is saying that we can call the amount of input x "a product", and therefore potentially an output, if, once exposed to the second input, thermal energy, satisfies the above at a pre specified point in time t. The constraint describes the qualitative change that the input must undergo to be considered a finished product. While the x input is still visible, the need for a different notation for the output has vanished. The energy input has apparently disappeared, but it is still present, as the exogenous source of thermal energy at temperature $T_1(t)$. As we are fully able to describe the qualitative transformation undergone by the input through the temperature coordinate $T(t, x, T_0, T_1)$, we no longer need to account for the output element $Y(t)$ which would appear in the neoclassical production function. The energy cost of transforming the marginal quantity of the raw material into finished product is given by Fourier's law (see below).

For additional clarity, we can describe the sequence of events which happen during the production process, in the time interval $t \in [-1, 1]$, characterizing inputs and outputs of the process. It is assumed for simplicity that $T_1(t)$ changes in a discrete fashion:

- $t = -1$: (INPUT) - A large amount of fluid, at the uniform temperature T_0, enters the process;

- $-1 \leq t < 0$: (INPUT) - The upper surface of the fluid in the production plant is in contact with a thermal energy source at a temperature $T_1 > k > T_0$. Thermal energy flows into the process;

- $t = 0$: (OUTPUT) - An amount of finished product, i.e. heated fluid, $x_0 \equiv x$ such that $T(t, x, T_0, T_1)|_{t=0} \geq k$ exits the process;

- $0 \leq t \leq 1$: (INPUT) - The upper surface of the fluid in the production plant is in contact with a thermal energy source at a new temperature $\hat{T}_1 > k$. Thermal energy keeps flowing into the process. At $t = 0$ the remaining fluid in the container is already at a temperature greater than T_0, described by the new initial condition $T_0(x; x_0)$. This new initial temperature of the fluid is non uniform. The initial temperature distribution depends on x_0, and decreases in the capacitor with depth; [2]

[2] At $t = 0$ the initial temperature of the fluid is not spatially uniform, but it is described by the temperature field already reached at $t = 0$, taking into account the removal of the amount of fluid

- $t = 1$: (OUTPUT) - An amount of finished product, i.e. heated fluid, $x_1 \equiv x$ such that $T(t, x, T'(x_0), \hat{T}_1)|_{t=1} = k$ exits the process; [3]

- $t = 1$: (OUTPUT) - An amount of waste, i.e. useless heated fluid, $w_1 \equiv x$ such that $T(t, x, T'(x_0), \hat{T}_1)|_{t=1} < k$ exits the process.

The steps of the production process may be visualized with the help of Figures 1 and 2.

This is a simple but fully explicit example of a technological description embedding a thermodynamic constraint. It exemplifies the Georgescu Roegen view of production as the qualitative transformation of a substance, which evolves into different good, with the production of entropy. Moreover, the thermodynamic limit of the production process is completely modeled. This contrast with other "thermodynamic" technologies proposed in the literature. The (Ruth, (1995)) model, for example, uses the neoclassical production function, in which the asymptotic behavior of the isoquant is interpreted as due, at each point in time, to a thermodynamic constraint, which may be generically time varying. The thermodynamic production technology adopted here instead models exactly why and how the thermodynamic efficiency varies over time. Only by adopting a technological description involving qualitative change we are able to model the transformation of one good into another, and compute the marginal cost of one good in term of the single production input. In the present technological description, as in the spirit of Georgescu Roegen theory, substitution possibilities between the two inputs, raw material and energy, are limited (i.e. it is possible to assume that the same finished product may be obtained starting from a different raw material (fluid), but it is less plausible to assume that there is a continuum of different fluids which will produce the same finished product).

2.3 Reconciliation with Neoclassical Production Frontier

Note that the condition "$x_1 \equiv x$ such that $T(t, x, T'(x_0), \hat{T}_1)|_{t=1} = k$" which defines the "product" at time $t = 1$ is an implicit constraint which links the amount which can be produced at $t = 1$ to the amount produced at $t = 0$. It describes a tradeoff between production at the two different dates, x_0 and x_1. This tradeoff arises because

x_0. This may be described by the function $T(t, x, T_0, T_1)$ with the x coordinate shifted by x_0,

$$T_0(x; x_0) \equiv T(t, x + x_0, T_0, T_1)\Big|_{t=0}$$

so that $T_0(x, ; x_0)|_{x=0} = T(t, x, T_0, T_1)|_{t=0,\, x=x_0}$. We use $T'(x_0)$ to indicate dependency on $\{x_0, T_0, T_1\}$. It is also possible $\hat{T}_1 = T_1$ See the Appendix for some insight.

[3]The production constraint at time $t = 1$ may be written:

$$T(1, x_1, T'(x_0), \hat{T}_1) \geq k\ . \tag{3}$$

At time $t = 1$ it is rational to use all the available production for consumption, and the constraint (3) holds as an equality.

the lower the amount of product which exits the process at $t = 0$, the more energy stays inside the process for a further period, enhancing future production.

Denoting $\bar{x}_0$ the maximum production achievable at $t = 0$, the difference $\bar{x}_0 - x_0$ represents "product" ready to exit the process, but which is kept inside the process in order to enhance future production.

The tradeoff between x_0 and x_1 represents the usual production frontier. This is the production process representation which is useful in a general equilibrium model. Through the implicit function theorem this defines the marginal cost, in terms of the produced good, of transforming good available at time $t = 0$ into good available at time $t = 1$.

However, as it will become apparent below, if we adopt the present technological description, we can also define an additional quantity: the marginal energy cost of transforming the raw input x into a qualitatively different finished product. In our technological description we can go one step ahead and provide the full technological description of how a cold fluid becomes a hot fluid, i.e. model the qualitative change of one of the inputs by means of the other (flow) input. This allows us to qualify the input x itself as output providing its quality has changed in the appropriate way. This allows to physically determine the energy transformation price of the economically useless input commodity X into finished product.

Adopting a classical thermodynamic approach, from Fourier'law (see for example Fuchs (1996), p.314) the instantaneous flux of thermal energy through the fluid at each depth is:

$$I_E(x) = -k_E \frac{\partial T(x)}{\partial x} \tag{4}$$

where k_E is the temperature coefficient of energy. $I_E(x)$ is decreasing in x and convex, and represents the energy price of the marginal unit of finished product. The energy cost resulting from the production model is clearly different from the fixed coefficients of the Leontief input-output energy analysis.
In order to simplify notation, set

$$\begin{aligned} T(x_0) &\equiv T(t, x_0, T_0, T_1)|_{t=0}, \qquad (5)\\ T(x_1, x_0) &\equiv T(t, x_1, T'(x_0), \hat{T}_1)|_{t=1}. \qquad (6) \end{aligned}$$

The production frontier $T(x_1, x_0) = k$ derived from (6), together with the condition $T(x_0) \geq k$ (which only bounds production achievable at time $t = 0$), would be the only aspect of the production function exploited in a neoclassical general equilibrium model in order to determine the inter temporal allocation of resources. The Appendix provides additional insight on (6). However, in the full description of the production process consistent with Georgescu Roegen theory developed in the previous section, (6) is only part of the story, and additional aspects of the production process are captured. In particular, an additional detail of the production process is here captured by (4). Thanks to (4), we can model, in general equilibrium the potential role of energy as numeraire and medium of exchange (money). The resulting market equilibrium will be markedly different from the neoclassical equilibrium, and will accommodate a correction for the externality due to waste.

3 Thermodynamic Production Technology and Equilibrium

We now embed the technological description of the production process in a simple general equilibrium model. The inter temporal resources allocation framework is based on the synthesis of neoclassical theory, as in Hirshleifer (1970). On top of this, an environmental externality is considered. In a neoclassical general equilibrium model in which agents consume at different dates, only the tradeoff between finished product available at different points in time, and not energy or entropy content, will matter. Some aspects of the full technological description of the production process achieved in the previous section, which includes the time dependence of production and the creation of waste, will have no relevance in the decision making process. Waste may indeed affect agents utility, but will be an externality that no individual agent will take into account. In this situation, departures from perfect market equilibrium, and ultimately central planning, will in principle be optimal. However, next it will be shown that, the additional condition (4), which plays no role in the barter economy model, may be used to determine a decentralized equilibrium in a monetary economy model in which a "useless" commodity is used as numeraire and means of payment, and that this market solution will have welfare properties similar to those obtained by central planning.

3.1 Welfare Optimality

In the economy the utility of each of the individuals, denoted $j = 1, \ldots, N$, is defined over consumption at time $t = 0$ and at time $t = 1$, denoted C_0^j, C_1^j respectively, and aggregate production waste, w_1. The utility function of individual j, $u^j(C_0^j, C_1^j, w_1)$ is a class $C^{(2)}$ function increasing and concave in C_0 an C_1, but decreasing in the waste externality w_1, which is outside the control of the individual. In the presence of externalities, welfare properties of market equilibrium may be improved upon by a social planner who can appropriately internalize the externality. However, in order to effectively correct this in an optimal way and improve welfare, the social planner must know $u_{w_1}^j$, every agent's marginal (dis)utility of waste. Alternatively, in a market framework, welfare optimality may be achieved through the use of taxes on waste, which would make the firm pay a price for the possibility to dump w_1 into the environment (or the fraction of w_1 which exceeds the environment receptivity). An optimal tax would be related to the consumers' marginal rate of substitution between waste and consumption. This would again be hard to identify in practice.

Under the technology described in the previous sections, waste is an unavoidable consequence of the production process. Entropic waste production is just another side of the production decision $\{x_0, x_1\}$. We can therefore write $w_1 = w(x_0, x_1)$, with partial derivatives $w_{x_0} > 0$, $w_{x_1} < 0$, where in equilibrium $x_0 = \sum_{j=1}^N C_0^j$, $x_1 = \sum_{j=1}^N C_1^j$. Under these conditions, both central planning and a tax on waste will lead to the same implication that, for welfare optimality, the absolute value of the marginal rate of transformation of x_0 into x_1 should be smaller than the marginal rate of substitution between C_0 and C_1:

$$-\frac{dx_1}{dx_0} < \frac{u^j_{C_0}}{u^j_{C_1}}\,, \tag{7}$$

where u_C denotes marginal utility. Neoclassical decentralized equilibrium will instead equate marginal rates of transformation and substitution through a price set in the market (See Section 3.2). Welfare improvement is achieved by decreasing aggregate consumption at $t = 0$, and increasing aggregate consumption at $t = 1$, as this decreases the aggregate amount of waste which negatively affects individuals' utility. We highlight below an alternative correction mechanism which would achieve the same result.

3.2 Decentralized Equilibrium under Energy Numeraire

Assume that, in this economy, thermal energy is solely assigned the role of numeraire and means of payment. This may be achieved by means of a Clower(1967) (cash in advance) constraint. That is, agents must secure before time $t = 0$ thermal energy (or, thermal energy rights) in advance of their trading.

A consumer must purchase the good from the firm with the money (energy) balances available before time $t = 0$, denoted $\overline{M}_j$. These balances represent a claim on energy of sufficient quality, i.e. suitable as input to the production process, which is available in the economy at time $t = 0$. We take money balances to be inside money.[4] Note that money (energy) is not an argument of the agent utility function[5] and therefore does not generate any satisfaction per se. Consumers cannot individually implement the production plan of the firm at both dates, but can make use of their initial endowment of energy at $t = 0$. Let P_e denote the spot price of the good in terms of energy. Consumer j, whose only endowment at time $t = 0$ is the other ways useless initial money balance $\overline{M}_j > 0$, spends it optimally by solving the problem:

$$\max_{C_0^j, C_1^j} \quad u^j(C_0^j, C_1^j, w_1)\,, \tag{8}$$

[4]Energy money balances may be external to the economy, held with a central banker (outside money), and although in principle could be physically delivered to the claim holder, in practice delivery may only occur at time $t = 0$ for a marginal (infinitesimal) amount. Alternatively, in a multi-period framework, $\overline{M}_j$ may represent the individual agent credit towards the firm, resulting from the previous period dividend on the share(s) of the firm owned, so that debits and credits of money balances net out in the economy (inside money). We take this latter interpretation. The distribution of inside money endowments across different agents will, in general, affect the equilibrium prices and allocations (see Gale, (1982)), except the special case of demand functions homogeneous of degree zero in prices and endowment. It is assumed that an aggregate amount of money which clears the market is distributed. This follows from the firm multi period maximizing behavior.

[5]Thermal energy might be an argument of the utility function, as in winter central heating of a house, but we are modeling here a fluid heated to produce a physical good.

$$
\begin{aligned}
sub \qquad & P_e\,(C_0^j + B\,C_1^j) \;\le\; \overline{M}_j\,, & (9)\\
& C_0^j \ge 0\,, & (10)\\
& C_1^j \ge 0\,, & (11)
\end{aligned}
$$

B being the real bond price. As the consumer is non satiated (9) holds as an equality. The first order condition of the consumer problem determine the bond price $B = \frac{u_{C_1}}{u_{C_0}}$. For this specific production technology the nominal price level, P_e, i.e. the spot price of good in terms of energy, may be determined by no arbitrage considerations, noting that the good produced is nothing but a (nonlinear) derivative on energy. It is clear that no consumer will pay more than the marginal cost defined by (4) for the quantity of good available at $t = 0$ purchased. If charged more, a consumer could simply short sell units of consumption and replicate them at lower cost with the energy received. Conversely, if charged less, additional free units could be produced with the available difference between energy price and energy cost. But even without resorting to the possibility that consumers may have to self produce a marginal unit of good at $t = 0$, the same equilibrium pricing would be achieved. A price P_e different from marginal energy cost cannot result in an equilibrium allocation. If consumers were charged more than the marginal energy price, they would in aggregate extract too little of the consumption good for the energy paid, leaving inside the process too much energy, which the firm will use to produce an additional amount of good over the following period. If they were charged less, they would buy too much good, leaving inside the the firm too little energy for it to carry out its production plan over the following period, implying a decrease in future production. In this economy the numeraire is also a factor of production at the margin. This determines a potential demand for money and allows the determination of the spot nominal price P_e.

The producer's problem is to maximize profit, which amounts to maximizing the value of production under the new numeraire (the value of the firm). As inputs are freely available, their cost need not be considered in the objective function. The problem is:

$$
\begin{aligned}
\max_{x_0,x_1} \qquad & x_0\ P_e + B\ x_1\ P_e\,, & (12)\\
sub \qquad & T(x_0) \ge k\ , & (13)\\
& T(x_1, x_0) \ge k\ , & (14)\\
& x_0 \ge 0\,, & (15)\\
& x_1 \ge 0\,. & (16)
\end{aligned}
$$

Clearing conditions require

$$
\sum_j C_0^j = x_0\,, \tag{17}
$$

$$
\sum_j C_1^j = x_1\,. \tag{18}
$$

Consider the problem (8)-(18). Under usual regularity conditions of the functions $u(.)$, only the constraint (9) of the consumer problem will be binding (no corner solution for consumption). With respect to the producer problem, we conjecture that only the constraint (14) will be binding, i.e. some of the production available at $t = 0$ will be reinvested to increase production at $t = 1$. We will later verify that this conjecture indeed holds. [6] [7]

Under the energy numeraire the price P_e may be set equal to the marginal energy cost of aggregate production $I_E(x_0)$. We can see this pricing function as the aggregate inverse demand function for the good available at $t = 0$. Another way of looking at this function is, to use derivative pricing terminology, to consider it the produced good "delta" on energy at the margin. Fourier's law (4), which is the basis for (1), will be the missing equation for the determination of the nominal price P_e in the problem (8)-(18). The explicit model for the (non constant) marginal energy cost provided by (4) allows closure of the general equilibrium model.

Consider the producer's problem under the new numeraire:

$$\max_{x_0,x_1} \; x_0 \; I_E(x_0) + B \; x_1 \; I_E(x_0),$$

$$sub \quad T(x_1, x_0) = k \; .$$

The producer's first order conditions under the new numeraire require:

$$\frac{dx_1}{dx_0} = -\frac{1}{B} + \frac{(x_0 + Bx_1)}{B\, I_E(x_0)} \frac{\partial}{\partial x_0} \left[k_E \frac{\partial T(x_0)}{\partial x_0} \right] \; . \tag{19}$$

On the other hand, the consumer's first order condition require

$$-\frac{u^j_{C_0}}{u^j_{C_1}} = -\frac{1}{B} \, . \tag{20}$$

From the heat equation (1) it is both necessary and sufficient that

$$\frac{\partial T(x_0)}{\partial t} = 0, \tag{21}$$

to go back to the barter economy equilibrium, in which

$$\frac{dx_1}{dx_0} = -\frac{1}{B} = -\frac{u^j_{C_0}}{u^j_{C_1}} \; . \tag{22}$$

[6] We may assume for simplicity that the N individuals may be aggregated into a representative individual.

[7] Note that, once the energy numeraire has been adopted, marginal energy cost of production is (globally) strictly decreasing (see Section 2.3), and therefore average cost is also decreasing. Therefore, if energy input cost had to be disbursed by the firm, the producer's decision, in a competitive market, would be not to produce. However, energy is freely available in nature, and has no opportunity cost. If it is not used to produce the "useful" good, it is wasted. The fact that more energy is spent to produce than it is recovered, is a characteristic of the thermodynamic production process. Still production needs to be undertaken, to generate utility for the consumer/shareholder. Production set to zero and the useless energy input distributed as a (higher) money dividend to the starving consumer is not an equilibrium.

In this limit case, which can only be achieved for $t \to \infty$, the constant price P_e will not affect the solution of the problem (8)-(18), and the single good will trade as if in a barter economy. However, as this model clearly shows, in the presence of an irreversible transformation this limiting steady state solution defys the second law of thermodynamics as it implies no increase in entropy. The general solution (19), which is the only one which can be achieved in our finite production interval, is instead in the Georgescu Roegen spirit. In the presence of entropy production, such solution will be closer to welfare optimality than the neoclassical solution (22).

4 Properties of the Decentralized Equilibrium

Some implications of the equilibrium are articulated below. [8] Looking at (19), we realize that the decentralized equilibrium under the energy numeraire will have the property (7) without recourse to environmental related taxation or central planning. However, in the decentralized equilibrium, this result is obtained within a fully fledged monetary economy in which the marginal rate of substitution u_{C_0}/u_{C_1} defines the (one plus) real interest rate ($(1+i_e) \equiv 1/B > 0$). If we define the (one plus) marginal rate of transformation, i_r, by $(1+i_r) \equiv -dx_1/dx_0 > 0$, we can write (19) as:

$$i_e = i_r + \frac{(x_0 + Bx_1)}{B\, I_E(x_0)} \, \frac{\partial}{\partial x_0} \left[k_E \frac{\partial T(x_0)}{\partial x_0} \right] \; . \tag{23}$$

For plausible temperature field models $T(x_0)$, the second term on the r.h.s. of (23) is positive for any finite t. The difference between i_e and i_r will be positive as long as there is a thermodynamic disequilibrium in the economy, i.e. $\partial T/\partial t \neq 0$. In such thermodynamic disequilibrium case, entropy is generated alongside the good production, and the real interest rate will reflect this. [9]

In absolute value, the marginal rate of substitution $(1+i_e)$ will be larger than the marginal rate of transformation $(1+i_r)$, as implied by (7), leading to an equilibrium closer to the one which would be imposed by a benevolent social planner. We just recalled that the slope $-1/B$ is the slope of the convex indifference curve of a representative investor evaluated at the equilibrium allocation. Therefore, the slope (19) will not be tangent to the indifference curve. The equilibrium allocation x_0 will be smaller than the value of x_0 under decentralized equilibrium in a barter economy, in which the indifference curve and production frontier are instead tangent to each

[8] It is clear that in order to achieve equilibrium in such economy no producer/consumer should be in a position to defy the cash in advance constraint, i.e. trade as if in a barter economy, or these agents would achieve a different marginal rate of transformation (like in the case of a self sustaining consumer/producer unit) and would generate a different market with a different interest rate, creating arbitrage opportunities (Roma, (2000)). That is, the choice of numeraire should not be left to agents.

[9] Note that the quantity of money necessary in the economy is $I_E(x_0)(x_0 + Bx_1)$. The second term on the r.h.s. of (23) is equal to the real quantity of money times the semi elasticity of the good spot money price with respect to production.

other. Consequently, x_1 will be larger. This is evident from Figure 3. It is clear also that even if (13) is binding in the barter economy, it will not be under the energy numeraire. This verifies the conjecture we made in Section 3.2. The energy numeraire has the effect of a redistribution of production across time, decreasing entropic waste.

Higher production today will increase waste production, and increase the real interest rate, lowering the price of consumption tomorrow. This will lead to a redistribution of production across time.

4.1 Waste Production

At $t = 1$ for $x > x_1$ we have some unfinished production, w_1, represented by fluid conveying low quality thermal energy, which has no economic use and goes wasted (into the environment). However, in the neoclassical equilibrium model, this waste, and its entropy content, is irrelevant in the decision making process of both consumers and producers, and need be corrected by a social planner or through environmental related taxation. The energy pricing mechanism makes decision making agents financially sensitive to this waste. It is clear from the preceeding discussion that waste will be smaller in the monetary economy in which energy is used as money. Also, its entropy content will be smaller. Consider in fact that the first order condition of the producer problem (19), which defines the inter temporal production plan of the firm, may be equivalently obtained from the dual problem

$$\min_{x_0,x_1} \quad -c\int_{x=x_1}^{\infty} \frac{\partial T(x_0,x)}{\partial x}\frac{1}{T(x_0,x)}dx$$

$$sub \quad x_0\ I_E(x_0) + B\ x_1\ I_E(x_0) = \bar{V}.$$

where the objective function is a measure of the entropy of the economically useless waste discarded at the end of the process, and $\bar{V}$ denotes a given market value of the production of the firm. The capacitor has in this model ideally infinite depth. [10] So, when firms maximize the value of their production under the energy numeraire, they are minimizing the amount of entropic waste produced. Therefore, if an entropy based measure of pollution is adopted, the pricing mechanism highlighted will minimize it.

5 Concluding Remarks

In this paper a stylized model of a technology fully consistent with the Georgescu Roegen view of the production process is used to exemplify some issues concerning the concepts equilibrium, entropy and waste in a neoclassical framework. In such a framework, these issues are elusive. The simple exercise proposed in this paper is to force economic agents to account in energy numeraire within such model, and

[10] The same result would hold for finite depth capacitor resting on a sink.

verify the consequences for the inter temporal equilibrium in the economy. The basic idea is to be able to measure the energy cost of aggregate production, through a detailed description of a production technology which is not in steady state with the consequent entropy production, and, at any point in time, does not allow for substitutions between inputs. The energy pricing of aggregate production is then integrated within marginalistic analysis. The model is admittedly quite simple, but sheds some light on how, by forcing the issue of the energy content on the valuations of economic agents, a market equilibrium may be altered towards a different production of entropic waste. This monetary arrangement leads to an equilibrium similar (but not identical) to that resulting from an environmental tax penalizing entropic waste. The resulting model has the features of a monetary economy, in which energy is used as money due to an institutional cash in advance constraint on agents. The model blends elements from marginalistic valuation theory and energetics, staying away from extreme interpretations within the theory of value. The issue of relative pricing of goods according to their energy content is avoided, but relevance is given in the valuation, beyond agents utility, to a service commodity, energy, that does enter the utility function, and that in the special context is defined as money. This may be less absurd if we take the historical perspective that commodity money has been the economic standard for thousands of years, and it is only some thirty years that the commodity backing has been lost. However, it is clear that any more ambitious model would have to take a firmer stand in respect of the theory of value adopted. The simple view of the aggregate economic process proposed may be the starting point for a more careful consideration of some of the modeling issues in the proper representation of thermodynamic aspects of production.

Appendix

We refer to the case in which the heat equation

$$\rho c \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[k_E \frac{\partial T(x,t)}{\partial x}\right], \; \rho, c, k_E > 0$$

takes the special form

$$\frac{\partial T(x,t)}{\partial t} = \sigma^2 \frac{\partial^2 T(x,t)}{\partial x^2} \tag{24}$$

where $\frac{k_E}{\rho c} = \sigma^2$. The initial condition is

$$T(x,0) = f(x) \tag{25}$$

with the additional condition

$$T(0,t) = \phi(t)\,. \tag{26}$$

The heat equation will be solved for $x \in [0, +\infty]$, assuming that the initial temperature has an initial distribution $f(x)$ and that the extreme $x = 0$ is maintained at the temperature (which may be time varying) $\phi(t)$.

The general solution is given by[11]

$$\begin{aligned} T(x,t) = & \frac{1}{2\sigma\sqrt{\pi t}} \int_0^\infty f(y)\left[e^{-\frac{(x-y)^2}{4\sigma^2 t}} - e^{-\frac{(x+y)^2}{4\sigma^2 t}}\right] dy + \\ & + \frac{x}{2\sigma\sqrt{\pi}} \int_0^t \frac{\phi(s)}{(t-s)^{\frac{3}{2}}} e^{-\frac{x^2}{4\sigma^2 (t-s)}}\, ds \end{aligned} \tag{27}$$

To verify the solution we can use the representation of the Dirac delta function given by

$$\delta(x) = \lim_{b \to +\infty} \sqrt{\frac{b}{\pi}}\, e^{-bx^2}\,.$$

For $f(y) = T_0$ and $\phi(t) = T_1$ (27) will take the form

$$\begin{aligned} T(x,t) = & \frac{1}{2}\left(T_0\, Erf(\frac{y-x}{2\,\sigma\,\sqrt{t}}) - T_0\, Erf(\frac{y+x}{2\,\sigma\,\sqrt{t}})\right)\Big|_{y \to +\infty} \\ & -\frac{1}{2}\left(T_0\, Erf(\frac{y-x}{2\,\sigma\,\sqrt{t}}) - T_0\, Erf(\frac{y+x}{2\,\sigma\,\sqrt{t}})\right)\Big|_{y=0} \\ & + T_1\, Erf(\frac{x}{2\,\sigma\,\sqrt{t-\tau}})\Big|_{\tau \to t} - T_1\, Erf(\frac{x}{2\,\sigma\,\sqrt{t-\tau}})\Big|_{\tau=0} \\ & = T_1 + (T_0 - T_1) Erf\left(\frac{x}{2\sigma\sqrt{t}}\right)\;. \end{aligned} \tag{28}$$

After a time interval $(t = t_0)$, (28) takes the form

[11] I thank Carlo Mari for pointing this out.

$$T(x) = T_1 + (T_0 - T_1) Erf\left(\frac{x}{2\sigma\sqrt{t_0}}\right) \; .$$

If the amount of fluid x_0 is removed, the temperature distribution is shifted to yield

$$f(y) = T_1 + (T_0 - T_1) Erf\left(\frac{y + x_0}{2\sigma\sqrt{t_0}}\right) \; . \tag{29}$$

The description of the temperature distribution (29) can then be used as the new initial condition $f(y)$ in the general solution (27), with the new condition $\phi(t) = \hat{T}_1$, and time elapsed described by t_1. The result is

$$T(x_0, x_1) = \hat{T}_1 + \frac{(T_0 - T_1)}{2\,\sigma\,\sqrt{\pi\,t_1}} \int_0^\infty \left(e^{\frac{-(x_1-y)^2}{4\,\sigma\,{t_1}^2}} - e^{\frac{-(x_1+y)^2}{4\,\sigma\,{t_1}^2}} \right) Erf\left(\frac{x_0 + y}{2\,\sigma\,\sqrt{t_0}}\right) dy$$

$$+ (T_1 - \hat{T}_1) Erf\left(\frac{x_1}{2\sigma\,\sqrt{t_1}}\right) \tag{30}$$

Setting

$$T(x_0, x_1) = k$$

for $t_1 = t_0 = 1$ defines the production frontier in our model.

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Figure 1

At time $t = -1$ the fluid in the container is at the temperature T_0. The upper surface is brought to the temperature $T_1 > T_0$. After a unit of time the fluid temperature in relation to its depth is $T(x)$, which is lower the deeper inside the container.

Period I

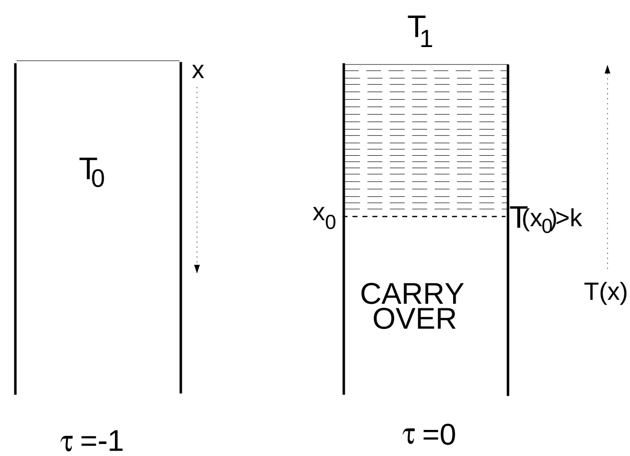

Figure 2

At time $t = 0$, after the removal of the produced amount x_0, the residual is at a (non uniform) higher temperature $T'(x_0) > T_0$, and will be carried over to the next period. Further heating until time $t = 1$ from the upper surface at the temperature T_1 results in the produced amount x_1, which has a marginal temperature equal to k. All the fluid below the depth x_1 is waste.

Period II

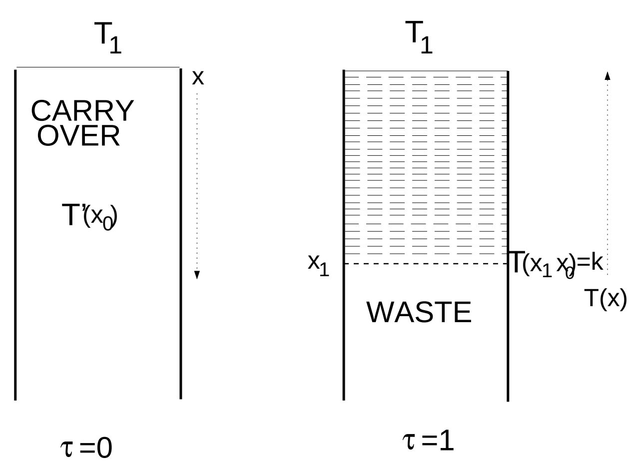

Figure 3

The Figure shows the difference between equilibrium in a neoclassical barter economy, in which the consumer convex indifference curve is tangent to the production frontier, and the equilibrium under the energy numeraire, in which the indifference curve is not tangent to the production frontier. The equilibrium allocation at time $t = 0$, x_0, is consequently moved to the left in the latter case.

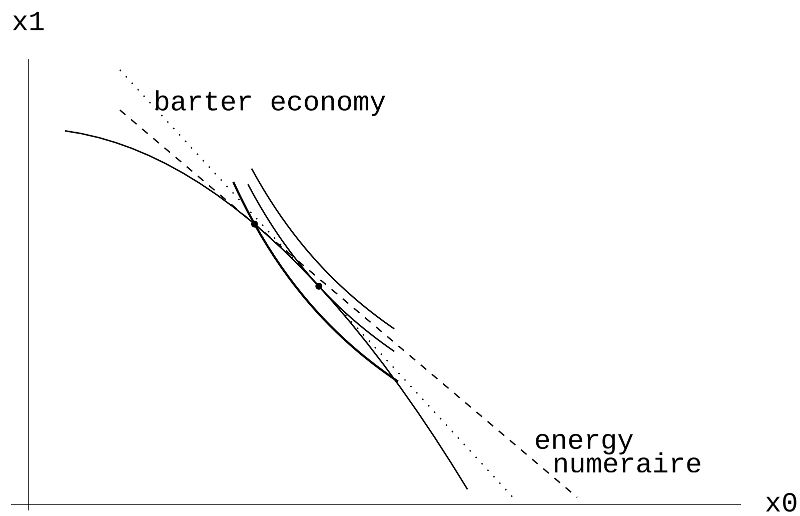

Figure 4

The Figure shows that under the energy numeraire in equilibrium the consumer indifference curve will be tangent to the budget constraint slope $-(1+i_e)$, and not to the production frontier.